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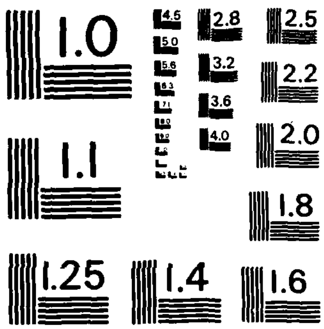

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research was undertaken on questions concerning the existence, uniqueness, continuous data dependence and numerical computations of solutions of various ill posed problems in partial differential equations. It was shown that a potential well theory is possible for certain hyperbolic problems in which a nonlinear boundary condition is prescribed and not possible in certain cases when the forcing term in the differential equation is singular. Several papers were accepted or submitted during this period Examples of titles are: "Inequalities between Dirichlet and Neumann eigenvalues", and "A potential well theory for the heat equation with a nonlinear boundary condition".

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Numerical Solution of Ill Posed Problems in Partial Differential Equations

(October 1, 1984 - September 30, 1985)

by

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I. Summary

This project is concerned with several questions concerning the existence, uniqueness, continuous data dependence and numerical computation of solutions of various ill posed problems in partial differential equations.

In the study of certain eigenvalue problems for the Laplacian on manifolds with boundary, several interesting, geometry dependent, inequalities between Dirichlet and Neumann eigenvalues were obtained. Numerical solution of a modified pendant drop equation demonstrated the existence of positive radial solutions of the equation in all of space.

It was shown that a potential well theory is possible for certain hyperbolic problems in which a nonlinear boundary condition is prescribed and not possible in certain cases when the forcing term in the differential equation is singular.

Also studied were several inverse problems for hyperbolic equations. Some new results for coefficient determination problems were obtained, which provide algorithms for computational purposes.

Several problems were studied which involved nonlinear parabolic equations. In one problem, numerical results were obtained for a semilinear heat equation with a convective term. In another, questions of uniqueness of solutions for nonlinear problems backward in time were examined.

II. Research Objectives

1. Eigenvalue Problems. For bounded domains in R^N, it is known that $\mu_k < \lambda_k$ where the μ_i, λ_i denote the Neumann and Dirichlet eigenvalues. We seek to improve this inequality under more restrictive conditions in the geometry.

2. Numerical Solution of Nonlinear Partial Differential Equations.

(a) We seek positive global solutions of a modified pendant drop equation in R^n:

$$\operatorname{div}\left(\frac{\operatorname{grad} u}{1 + |\operatorname{grad} u|^2}\right) + \lambda u^q - u = 0 \quad (\lambda>0,\ q>1).$$

in R_n, with the "boundary condition" $u(x) \to 0$ as $|x| \to \infty$. The question of the existence of such solutions was raised by J. Serrin.

(b). We consider the solution of an initial-boundary value problem for a nonlinear heat equation with convection:

$$u_t = u_{xx} + \epsilon\, g(u)u_x + f(u) \qquad (\epsilon > 0)$$

with particuliar attention to the cases $g(u) = u$, $f(u) = |u|^{p-1}u$, $p>1$. We are concerned with the long time behavior of such solutions.

(c). We consider, in several space dimensions, an initial-boundary value problem for the wave equation with a singular nonlinearity:

$$u_{tt} = \Delta_n u + \epsilon\,(1 - u)^{-\beta}, \quad (\beta>0,\ \epsilon > 0).$$

we seek conditions on ϵ and the initial data for which the solution remains less than one (pointwise) for all time. The problem is complicated by the lack of an embedding inequality from H_0^1 into L^∞ for more than one space dimension.

3. Nonlinear Hyperbolic Problems (theory). In 1974, L.E. Payne and D.H. Sattinger developed a potential well theory and a corresponding existence - nonexistence theorem for initial-boundary value problems of the form

$$u_{tt} = \Delta_n u + f(u)$$

where (i) f is either a convex point function or (ii) convex for $u>0$, concave for $u<0$ and grows like $|u|^p$ for $p>1$ at infinity. The arguments in the second case were seriously flawed. We consider the problem in this case, as well as the question of developing analogous results for a similar problem when the equation is linear but when $\partial u/\partial n = f(u)$ on a portion of the boundary of the spatial domain. We seek similar results for parabolic problems ([2], [3]).

4. Hyperbolic Inverse Problems. The overall goal is to understand the relationship between coefficients and boundary values for linear hyperbolic equations and systems. Depending on what side conditions are are imposed a large variety of such problems can be formulated. We are interested in (i) the mathematical structure of the associated mappings and (ii) development of workable algorithms for determing coefficients from measurements of boundary values.

5. Nonlinear Parabolic Equations. The objective is to understand in as much detail as possible the dynamical behavior of systems governed by nonlinear parabolic equations and systems. Of interest to us currently are some questions involved in the study of final value problems, e.g. existence, uniqueness and continuous dependence estimates for solutions of parabolic equations backwards in time. We believe that the answers to the questions will be important in the further development of control theory for systems governed by nonlinear parabolic equations.

III. Status of Research

(References refer to the publication list attached (V)).

1. Eigenvalue problems. We have shown (with H.F. Weinberger) that if the domain is convex with a $C^{2+\alpha}$ boundary, then $\mu_{k+N} < \lambda_k$, $k = 1, \ldots$ If the mean curvature is positive then $\mu_{k+1} < \lambda_k$ (Aviles). Also, if, at every point of the boundary, every sum of length $N - R + 1$ of the set of numbers $\{\kappa_1, \ldots, \kappa_{N-1}, (N-1) H\}$ is nonnegative, then $\mu_{\kappa+R} < \lambda_\kappa$. (Here the κ's are the principal curvatures and H is the mean curvature.) Examples and counterexamples are given. Also, a characterization in terms of the equations defining the boundary surface is given. ([1].)

2. Numerical Solution of Nonlinear Partial Differential Equatins.

(a) It was shown in [7], numerically, that radial, global, positive solutions of the equation given in II. 2. a above exist for all λ sufficiently large and $1<q<(n+2)/(n-2)$. This was a surprise to Serrin who (with Ni) had shown that no global solutions existed for all $\lambda<\lambda_q = (1/2\,(q-1)/(q+1))^{1/2\,(q-1)}$ and $q<(n+2)/(n-2)$. The computations motivated Serrin and Peletier to prove the existence of such solutions. The computations also show that the nonexistence result is not best possible. It was also shown in [7] that global, radial solutions which change sign also exist. (Serrin and Peletier have not shown this theoretically.) The computations raised other interesting questions about the nature of the solutions of this equation.

(b) We considered the time independent equation

$$u_{xx} + (g(u))_x + f(u) = 0, \ 0<x<1$$

where $g(u) = \frac{1}{2} \varepsilon u^2$ and $f(u) = |u|^{p-1}u, \ p>1$.

The numerical experiments were performed for various ε and p. When $\varepsilon = 0$, one can easily that there is one and only one non-zero positive solution for $p > 1$. When $\varepsilon \neq 0$, the results varied depending on p. In the case $1 < p < 3$, there are two positive solutions for ε in certain finite interval and there is no positive solution for ε outside that finite interval. In the case $p = 3$, there is exactly one solution for all ε in a bounded interval, while when $p > 3$, numerical results showed that there is exactly one non-zero positive solution for any ε. Having discovered the correct results, in some cases, we have been able to verify them analytically.

We are now working on the time dependent problems with various initial inputs.

(c) The first initial-boundary value problem for the equation in II.2.c. above was shown to have a unique local (in time) solution for all $\varepsilon > 0$, $p > 0$ in dimensions $n = 1,2,3$ under appropriate conditions on the initial data and geometry. The solution can be continued as long as $u < 1$. It has been shown that a potential well theory is not possible for this problem in H_0^1 for $n = 2,3$. However, a certain a priori inequality for solutions guarentees global existence via energy considerations. Numerical evidence indicates that such an a priori inequality holds when $n = 2,3$ and the solution is global. ([5]). (The case $n = 1$ was considered earlier.)

3. Nonlinear Hyperbolic problems (Theory). A potential energy well theory applies for solutions of these problems when the nonlinearity is of the form given in II.3. above. Each problem has a global weak solutions provided the data lies in the potential well and the total initial energy is small. The global solution is obtained as an expansion in normal modes in terms of the Helmholtz eigenfunctions and the eigenfunctions for a modified Steklov problem. The proof of global existence is valid for all potential wells of positive depth and all dimensions $n > 1$, and cam be used to generalize Sattinger's theory.

Solutions of these problems which start in a region exterior to the potential well with sufficiently small total initial energy can only exist for a finite time. The proof of this fact requires stronger hypotheses on f and corrects a key lemma of Payne and Sattinger.

4. Hyperbolic Inverse Problems. We have derived, analyzed, and done numerical experiments on some algorithms for solving inverse problems for layered elastic and acoustic media. We have also studied some related otimization problems. The behavior of these algorithms has been encouraging in the special cases that we have tried so far. We are now in the process of trying to handle more complicated situations. The analysis which suggested these methods has also been used to prove uniqueness theorems for the problem of determining coefficients in a differential equation from measurement of certain boundary values. ([8, 9].)

5. Nonlinear parabolic problems. For a class of nonlinear parabolic problems including e.g. the porous medium equation, we have established a certain alternative theorem, describing the behavior of nonnegative solutions near $t = 0$. This result has implications for the question of what sort of initial values a from which solution may arise. ([10].)

We have also proven fairly general results on the continuous dependence on nonlinearities in parabolic and elliptic problems. Aside from their own intrinsic interest, results of this type are necessary as technical tools in the study of backward parabolic problems. The arguments used here also improve on known convergence results for penalization approximations to solutions of certain types of variational inequalities.

IV Publications

1. H.A. Levine and H.F. Weinberger, Inequalities between Dirichlet and Neumann Eigenvalues, Arch. Rat. Mech. Anal. (in print)

2. H.A. Levine and R.A. Smith, A Potential Well Theory for the Wave Equation with a Nonlinear Boundary Condition, J. Reine Ang. Mat. (submitted)

3. H.A. Levine and R.A. Smith, A Potential Well Theory for the Heat Equation with a Nonlinear Boundary Condition, Math. Meth. Appl. Sci. (in print).

4. R.A. Smith, Theoretical and Numerical Studies of some ill-posed problems in Partial Differential Equations, Ph.D. dissertation, Iowa State University, October 18, 1985.

5. R.A. Smith, On a Hyperbolic Quenching Problem in Several Dimensions, (manuscript in preparation).

6. H.A. Levine and R.A. Smith, "A Potential Well Theory for the Wave Equation with a Nonlinear Boundary Condition," Proceedings of Int. Conf. on Theory and Applications of Differential Equations (in press).

7. T.K. Evers, Numerical Search for Ground State Solutions of a Modified Pendant Drop Equation, Masters Paper, Iowa State University, October, 1985.

8. P.E. Sacks and W. Symes, Recovery of the Elastic Coefficients of a Layered Half Space. (in preparation).

9. P.E. Sacks and F. Santosa, A. Simple Computational Scheme for Determining the Sound Speed of an Acoustic Medium from its Surface Impulse Response, SIAM J. Scient. and Stat. Comp. (submitted).

10. P.E. Sacks, Behavior near t = 0 for a Class of Nonlinear Diffusion Equations, Proc. I.C.T.P. Autumn Workshop on Semigroup Theory and Applications, Pitman Pub.

11. T.S. Chen, On Least Squares Approximation to Compressible Flow Problem, Computers and Mathematics with Applications (in press).

V. Personnel

A. Senior Staff

1. Professor Howard A. Levine (P.I.) — 3 months
2. Professor Paul E. Sacks — 3 months
3. Professor Tsu Fen Chen — 2 months

B. Graduate Research Assistants

1. Richard A. Smith — 12 months

   (Smith will defend his Ph.D. dissertative November 19, 1985)

   Title: Theoretical and Numerical STudies of Some Ill Posed Problems in Partial Differential Equations.

2. Thomas K. Evers — 3 months

   (Evers defended his M.S. thesis October 18, 1985).

   Title: Numerical Search for Ground State Solutions of a Modified Capillary Equation.

(Both students will receive December, 1985 degrees, assuming successful defenses of their work).

VI. Interactions

A. Seminar, Colloquim Speakers

1. Professor Ralph Showalter (University of Texas, Austin) 10/2/84
2. Professor Brian Straughan (University of Wyoming) 10/23/84
3. Professor Murray H. Protter (University of California, Berkeley) 4/8/85
4. Professor Philip S. Crooke (Vanderbilt University) 4/23/85

B. Other Interactions

1. Howard A. Levine

   (a) Consulted with L.E. Payne at Cornell University June 22-29, 1985 and held informal discussions at the AFOSR Mathematics and Information Sciences, July, 1985.

2. Paul E. Sacks

(a) Consulted at Cornell University with F. Santosa, W. Symes and others on inverse problems June 24-July 5, 1985.

(b) Consulted at Rice University with W. Symes on inverse problems, July 30-August 16, 1985

3. Tsu F. Chen

(a) Spoke at 1985 SIAM meeting in Pittsburg June 22-26, 1985, on least squares approximations to compressible flow problems. She also consulted with G. Fix at Carnegie-Meldon.

4. Richard A. Smith

(a) Presented a paper on his dissertation at the Internatinal Conference on the Theory and Applications at Pan American University, Edinburg, Texas, May 20-23, 1983.

(b) Spoke on "Some ill-posed problems in partial differential equations," Lawrence Livermore National Laboratory, Livermore, California July 25, 1985.

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